D-003-001616

Seat No. ________

B. Sc. (Sem. VI) (CBCS) Examination

April / May – 2015

Mathematics : BSMT-601(A)

(Graph Theory & Complex Analysis - II)

Faculty Code : 003

Subject Code : 001616

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (i) All questions are compulsory.

(ii) Answer all MCQ in answerbook only.

1 Attempt all M.C.Q. : **20**

(1) An isolated vertex in a graph G has degree

(A) 1 (B) 0

(C) 2 (D) None of these

(2) The maximum number of edges in a simple graph with 4 vertices is

(A) 3 (B) 6

(C) 5 (D) None of these

(3) The total number of edges in a complete graph with n vertices is

(A) $\frac{n(n+1)}{2}$ (B) $\frac{n(n-1)}{2}$

(C) $\frac{n^2(n+1)}{2}$ (D) None of these

(4) The number of vertices in a binary tree is always

(A) zero (B) odd

(C) even (D) None of these

(5) What is the chromatic number of complete graph with n vertices ?

(A) $n-1$ (B) $n+1$

(C) n (D) None of these

(6) Nullity of a connected graph with n vertices and e edges is

(A) $e+n-1$ (B) $e+n+1$

(C) $e-n+1$ (D) None of these

(7) Every tree with two or more vertices is

(A) 1-chromatic (B) 3-chromatic

(C) 2-chromatic (D) None of these

(8) The rank of an incidence matrix of a connected graph G with 4 vertices is

(A) 1 (B) 2

(C) 3 (D) 4

(9) Regions of a connected planar graph with 4 vertices and 6 edges is

(A) 1 (B) 2

(C) 3 (D) 4

(10) The number of pendant vertices in any binary tree with 23 vertices are

(A) 11 (B) 13

(C) 14 (D) 12

(11) The sum function of the series $\sum \frac{z^n}{n!}$ is

(A) Sine function (B) Cosine function

(C) Logarithmic function (D) Exponential function

(12) Radius of convergence of infinite series $\sum \left(1+\frac{1}{n}\right)^{n^2} Z^n$ is

(A) 0 (B) ∞

(C) e (D) $\frac{1}{e}$

(13) The image of circle $|z-1|=1$ in the complex plane under the mapping $w=u+iv=\frac{1}{z}$ is

(A) $u=\frac{1}{2}$ (B) $v=\frac{1}{2}$

(C) $|w-1|=1$ (D) $u^2+v^2=1$

(14) The region $|z|>1$ represent

(A) exterior of unit disk (B) closed unit disk

(C) open unit disk (D) None of these

(15) The fixed point of the mapping $w=\frac{3iz+13}{z-3i}$ are

(A) $3i\pm 2$ (B) $3\pm 2i$

(C) $2\pm 3i$ (D) $-2\pm 3i$

(16) A pole of order of $f(z)=\frac{1-e^{2z}}{z^3}$ at $z=0$ is

(A) 2 (B) 3

(C) 0 (D) None of these

(17) The residue of a function can be evaluated only if the pole is an isolated singularity

(A) true (B) false

(C) partially false (D) None of these

(18) $\operatorname{Re}s\left(\tan z, \frac{\pi}{2}\right)$ is

(A) 1 (B) –1

(C) 0 (D) None of these

(19) Residue of $f(z)$ at a simple pole $z=a$ is

(A) $\lim_{z\to a} z f(z)$ (B) $\lim_{z\to a} \frac{f(z)}{z-a}$

(C) $\lim_{z\to a} (z-a) f(z)$ (D) $\lim_{z\to a} \frac{z-a}{f(z)}$

(20) To evaluate the integral of the type $\int_0^{2\pi} \phi(\cos\theta, \sin\theta)\, d\theta$

the contour used is

(A) any circle (B) semicircle

(C) rectangle (D) unit circle

2 (a) Attempt any three : **6**

(i) Define : Finite graph, Isolated vertex.

(ii) In complete graph k_n the number of edges are 300 then obtain the number of vertices.

(iii) Define : Path matrix.

(iv) Obtain Rank and nullity for complete graph.

(v) Prove that a binary tree with n vertices has $\frac{n+1}{2}$ pendent vertices.

(vi) If G is a simple connected regular graph with e edges and f regions then prove that $e \geq \frac{3}{2} f$.

(b) Attempt any three : **9**

(1) State and prove graph theory's first theorem.

(2) Prove that a simple graph with n vertices and k component can have atmost $(n-k)(n-k+1)/2$ edges.

(3) Prove that $k_{3,3}$ is non-planner graph.

(4) Define Adjacency matrix and state its properties.

(5) Define : Tree and Spanning tree.

(6) Prove that a graph with atleast one edge is 2-chromatic if and only if it has no circuits of odd length.

(c) Attempt any two : **10**

(1) Explain Konigsberg bridge problem and the solution given by Euler.

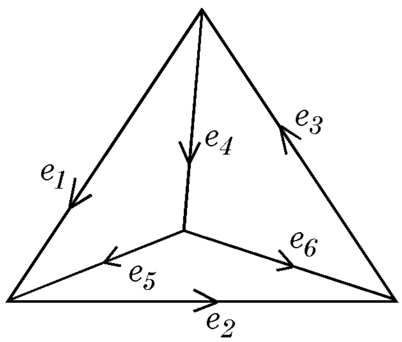
(2) State and prove necessary and sufficient condition for a graph to be disconnected.

(3) Define cut-set vector and in usual notation prove that $(w_s, \oplus, ')$ is a subspace of W_G over field $GF_2 = \{0, 1\}$.

(4) Prove that a vertex V in a connected graph G is a cut-vertex if and only if there exist two vertices x and y in G such that every path between x and y passes through V.

(5) Define : Minimal Decyclization.

For the following graph G, find minimal decyclization.



3 (a) Attempt any three : **6**

(1) Expand e^2 by Taylor's series about $z = 1$.

(2) Show that $x + y = 2$ transform into the parabola $y^2 = -8(v-2)$ under the transformation $w = z^2$.

(3) Prove that when $z \neq 0$

$$\frac{\sin\left(z^2\right)}{z^4} = \frac{1}{z^2} - \frac{z^2}{3!} + \frac{z^6}{5!} - \frac{z^{10}}{7!} + \ldots\ldots.$$

(4) Find residue and pole of order of the function $\frac{\sinh z}{z^4}$.

(5) Define : Mobius transformation.

(6) Define : Isolated singular point. Find isolated singular point of the function $\frac{z+1}{z^3\left(z^2+1\right)}$.

(b) Attempt any three : **9**

(1) Expand $\sinh z$ in power of $z-\pi i$.

Prove that $\lim_{z\to\pi i}\frac{\sinh z}{z-\pi i}=-1$.

(2) Expand $f(z)=\frac{z}{(z-1)(z-3)}$ into Laurent's series for $0<|z-1|<2$.

(3) Prove that the transformation $w=2z+z^2$ maps the unit circle $|z|=1$ of z-plane into a cardiod in w-plane.

(4) Find the value of integral $\int_c \frac{dz}{z^3(z+4)}$ where $c:|z|=2$.

(5) Show that the composition of two bilinear maps is again a bilinear map.

(6) Prove that $\operatorname{Res}_{z=i}\frac{z^{1/2}}{\left(z^2+1\right)^2}=\frac{1-i}{8\sqrt{2}}$ where $|z|>0$

$0<\arg z<2\pi$.

(c) Attempt any two : **10**

(1) State and prove Taylor's infinite series for an analytic function.

(2) Prove that the transformation $(w+1)^2 = \frac{4}{z}$ transform the unit circle of w-plane into the parabola of z-plane.

(3) State and prove Cauchy's residue theorem.

(4) Prove by using Cauchy residue theorem

$$\int_0^{2\pi} \frac{d\theta}{1+K\sin\theta} = \frac{2\pi}{\sqrt{1-K^2}} \text{ where } K^2 > 1$$

(5) Prove by using Cauchy residue theorem

$$\int_0^{\infty} \frac{x^2 dx}{\left(x^2+4\right)\left(x^2+4\right)^2} = \frac{\pi}{200}.$$
